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MODELING THE SPREAD OF AN OIL SLICK ON THE SEA SURFACE

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The spread of an oil slick in the sea is considered. A mathematical model of the process is proposed and a formulas for determining the oil slick sizes in explicit form are obtained.

Key words: *oil slick, self-similar solution, mathematical modeling of spread.*

Introduction. In recent years, there has been increased interest in the mechanism of oil spread in sea water. The total amount of petroleum products entering the ocean is proportional, as a first approximation, to the world oil production, creating an increasing danger of marine pollution. The basic sources of marine pollution by petroleum products are sea oil production, marine transportation of oil and its associated operations, transfer of petroleum products by rivers, oil supply from the atmosphere and industrial and household sources, and natural oil infiltration at sea coasts with great bottom sediment thicknesses and tectonic activity [1]. The volume of the petroleum products entering the oceans due to accidents of tankers and other vehicles is an insignificant part of the total pollution. However, pollutions of this type are the most dangerous since they cannot be predicted and are of a local nature and high power, which complicates self-purification and can result in a disturbance of ecological balance.

In planning and performing work for eliminating accidental oil spills in the ocean, it is necessary to predict the oil spread on sea water. Such predictions can be used, in particular, to warn of the danger of oil pollution of the coastal zone, the passage of an oil spill through areas of intense economic activity, vehicle paths, etc.

Oil spread in the sea during accidental spills is a complex process, whose description requires accounting for a large number of various factors. For the case of an instantaneous local spillage of a certain volume of oil, this process can be imagined as follows. Oil first spreads on the sea surface under the action of forces due to gravity (the density of water is higher than the oil density, and oil is therefore elevated above the sea surface) and viscous friction, and, then, the oil spread is due to the surface tension force. The problem is complicated by the fact that, during spread, the properties of oil change due to its evaporation and dissolution in water. At a certain time, the work of the surface tension force changes sign and the spread stops. Further increase in the slick size is determined by turbulent wind and current, i.e., turbulent diffusion. In addition to the mechanism of spread of an oil spill relative to its center of gravity, it is important to study the drift of the oil spill under the action of wind, current, and surface waves.

Approaches to Modeling the Spread of an Oil Slick. At present, there are a great number of approaches to modeling the spread of an oil slick on the sea surface. In [2, 3], the spread of an oil slick on the sea surface is represented in the form of a sequence of three phases: inertial, gravitational-viscous, and surface-tension phases. Based on this simplification of the spread, a method is proposed to determine the time dependence of the radius of an oil slick for each phase. For the inertial phase, this dependence is determined from the approximate equality of the forces due to the horizontal pressure gradient and inertia, for the gravitational-viscous phase, it is determined from the approximate equality of the forces due to the horizontal pressure gradient and viscosity, and for the surface tension phase, from the approximate equality of the forces due to viscosity and surface tension.

It should be noted that the proposed method is fairly simple but it has a number of significant drawbacks, in particular, a simplified treatment of the main mechanisms involved in oil spread, the absence of an accurate criterion for determining the duration of the different phases of the spread, etc.

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In previous studies [4–6], two- and the three-dimensional hydrodynamic models consisting of systems of nonlinear partial differential equations have been proposed to study the spread of an oil slick on the sea surface. However, the solution of these equations is very complicated because of their nonlinearity and the presence of a free boundary. Therefore, there is a need to develop a simple mathematical model for the main mechanisms of oil spill spread in the sea that is suitable for practical use. The most promising approach is the one in which the basic parameters and equations are averaged over the slick thickness.

Formulation of the Problem and Method of Solution. We consider the spread of an oil slick resulting from an accidental oil spillage in the sea. The spread of the oil slick on the sea surface is assumed to affected by the forces due to gravity and viscous friction. The main characteristics of the oil slick are considered to be its radius and thickness. Then, using the mass conservation equation for the elementary volume of the oil slick and the equation of motion, the mathematical model of the examined process in the axisymmetric case can be represented in the form of the mass conservation equation

$$\frac{\partial h}{\partial t} + \frac{1}{r}\,\frac{\partial\,(ruh)}{\partial r} = 0 \tag{1}$$

the equation of motion

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} = -g\delta\,\frac{\partial h}{\partial r} - \frac{\tau}{\rho_{\rm oil}h}, \tag{2}$$

where h is the thickness of the oil slick, u is the velocity of motion averaged over the slick thickness, τ is the tangential stress on the lower boundary of the slick, g is the acceleration due to gravity, $\delta = (\rho_{\rm w} - \rho_{\rm oil})\rho_{\rm oil}^{-1}$, $\rho_{\rm w}$ and $\rho_{\rm oil}$ are the density of water and oil, respectively, r is the radial coordinate, and t is time.

Since the spread of an oil slick is a slow process, we can ignore the acceleration of motion and simplify the equation of motion. Then, setting

$$\tau = \mu u/h,$$

from Eq. (2) we obtain

$$u = -\frac{\rho_{\rm oil}\delta g h^2}{\mu}\,\frac{\partial h}{\partial r},$$

where μ is the oil viscosity. Substituting the obtained expression for u into Eq. (1), we have

$$\frac{\partial h}{\partial t} = \frac{\alpha}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial h^4}{\partial r}\Big), \tag{3}$$

where $\alpha = \rho_{\rm oil}\delta g/(4\mu)$.

The initial spread of the slick is represented in the form of a concentrated instantaneous source:

$$h(r,0) = 0 \quad \text{at} \quad r \neq 0, \qquad 2\pi\int_0^\infty h(r,0)r\,dr = V_0 \tag{4}$$

(V_0 is the initial volume of spilled oil). The boundary condition is given by

$$h(\infty,t) = 0, \qquad t \geqslant 0. \tag{5}$$

The analysis of dimensions performed in [7] shows that the solution of problem (3)–(5) is self-similar and depends on the quantities t, r, α, and V_0. The solution of problem (3)–(5) has the form

$$h(r,t) = \psi(t)f(\xi), \qquad \xi = r/\varphi(t). \tag{6}$$

Substitution of expressions (6) into Eq. (3) yields

$$\frac{\varphi^2 f}{\alpha\psi^4}\,\frac{d\psi}{dt} - \frac{\varphi'\varphi\xi}{\alpha\psi^3}\,\frac{df}{d\xi} = \frac{1}{\xi}\,\frac{d}{d\xi}\Big(\xi\,\frac{df^4}{d\xi}\Big). \tag{7}$$

From the analysis of dimensions, it follows that

$$\psi(t) = \Big(\frac{V_0}{2\pi\alpha t}\Big)^{1/4}, \qquad \varphi(t) = \Big(\frac{V_0^3\alpha t}{8\pi^3}\Big)^{1/8}.$$

From Eq. (7) we obtain an ordinary differential equation for the function $f(\xi)$:

$$\frac{d^2 f^4}{d\xi^2} + \frac{1}{\xi}\frac{df^4}{d\xi} + \frac{\xi}{8}\frac{df}{d\xi} + \frac{f}{4} = 0. \tag{8}$$

Multiplying both sides of Eq. (8) by the quantity ξ, we obtain the following equation in complete differentials:

$$\frac{d}{d\xi}\Big(\xi\frac{df^4}{d\xi} + \frac{\xi^2}{8}f\Big) = 0. \tag{9}$$

Relation (4) implies that

$$2\pi\int_0^\infty h(r,t)r\,dr = \text{const} = V_0 \tag{10}$$

for any value of t. In view of conditions (4) and (6), from expression (10) we obtain

$$\int_0^\infty \xi f(\xi)\,d\xi = 1, \qquad f(\infty) = 0. \tag{11}$$

Integration of Eq. (9) on the interval $[0,\xi]$ yields the relation

$$\xi\frac{df^4}{d\xi} + \frac{\xi^2}{8}f = 0,$$

and integration of the above relation taking into account the second condition (11) yields the general solution

$$f(\xi) = \begin{cases} (\sqrt[3]{3}/4)(\xi_0^2 - \xi^2)^{1/3}, & \xi \leqslant \xi_0, \\ 0, & \xi \geqslant \xi_0. \end{cases}$$

Using the first condition (11), we determine the constant ξ_0:

$$\frac{\sqrt[3]{3}}{4}\int_0^{\xi_0} \xi(\xi_0^2 - \xi^2)^{1/3}\,d\xi = 1.$$

From this, we have $\xi_0 = 4/\sqrt[8]{162}$.

Thus, the solution of problem (3)–(5) can be represented in the final form

$$h(t) = \begin{cases} \dfrac{\sqrt[3]{3}}{4}\Big(\dfrac{V_0}{2\pi\alpha t}\Big)^{1/4}\Big(\xi_0^2 - \dfrac{r^2}{(V_0^3\alpha t/(8\pi^3))^{1/4}}\Big)^{1/3}, & r \leqslant r_k(t), \\ 0, & r \geqslant r_k(t). \end{cases} \tag{12}$$

From solution (12), it follows that, at each time, the oil slick has finite radius $r_k(t)$ given by the relation

$$r_k(t) = \xi_0\Big(\frac{V_0^3\alpha t}{8\pi^3}\Big)^{1/8}. \tag{13}$$

For an oil slick of the maximum thickness, we obtain the expression

$$h(0,t) = \frac{\sqrt[3]{3}}{4}\Big(\frac{V_0}{2\pi\alpha t}\Big)^{1/4}\xi_0^{2/3}. \tag{14}$$

Thus, formulas (12)–(14) can be used to predict the sizes and locations of oil slicks during accidental oil spills in the ocean.

REFERENCES

1. S. A. Gerlach, *Marine Pollution*, Springer-Veralg, New York (1981).
2. J. A. Fay, “Physical processes in the spread of oil on a water surface,” in: *Proc. of the Joint Conf. Prevention and Control Oil Spills*, Amer. Petrol. Inst., Washington (1971), pp. 463–467.

3. J. A. Fay, *The Spread of Oil Slicks on a Calm Sea,* Plenum Press, New York (1969), pp. 53–63.
4. A. S. Monin and P. V. Krasitskii, *Phenomena on the Ocean Surface* [in Russian], Gidrometeoizdat, Leningrad (1985).
5. R. V. Ozmidov, *Impurity Diffusion in the Ocean* [in Russian], Gidrometeoizdat, Leningrad (1985).
6. L. N. Karlin amd S. B. Asadov, "Modeling of pollution from accidental oil spills in the Caspian sea," in: *The Int. and Region. Importance of the Baku–Tbilisi–Ceyhan Pipeline*, Proc. Int. Conf. (Baku, June 6–7, 2002), Adilogly, Baku (2002), pp. 110–112.
7. G. I. Barenblatt, *Similarity, Self-Similarity, and Intermediate Asymptotics* [in Russian], Gidrometeoizdat, Leningrad [in Russian] (1978).